11 - 16 - 05

Review

Today	BH	103	1 - 3 PM
Tomorrow	ECH	110	10 - Noon.

Ch 9 HW Exercises

- p 213 - 15 # 2(all), 5 all) 8a,b
- Handout.

Due Mon after Thanksgiving.

x = prey
y = predator

Which of the following is true in a Lotka-Volterra predator prey model:

$\frac{\Delta x}{\Delta t} = ax - (by)x$
+births − deaths

$\frac{\Delta y}{\Delta t} = cxy - dy$

A. The prey birth rate is constant.
B. The predator birth rate depends on the # of prey.
C. The prey death rate depends on the # of predators.
D. The predator death rate depends on the # of prey.

pred deaths = d is constant.

$\frac{ax}{x}$ = # prey births / # prey

$\frac{cxy}{y}$ = # pred births / # pred

What is the name of points where neither species changes quantity?

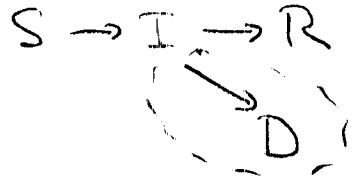
A. Poincare Points
B. Equilibrium Points
C. Null Points
D. Cooperative Points

Species r and s are represented in a phase plane with r on the horizontal axis and s on the vertical axis. In a portion of the phase plane, $\Delta r > 0$ and $\Delta s < 0$. In that portion of the phase plane

$\Delta r > 0$ → r increases; $\Delta s < 0$ → s decreases

A. r increases and s increases.
B. r increases and s decreases.
C. r decreases and s increases.
D. r decreases and s decreases.

The basic SIR model for measles is modified to allow for the possibility that the measles are fatal. Which is true about the change in the diagram for the model?



A. A new group is added to the model but no arrows
B. A new arrow is added but no new groups
C. Both a new arrow and a new group are added
D. The diagram is unchanged.

A group of medical doctors researched ways to increase handwashing by children. They believe they can cut in half the ~~chance~~ children's chance of infection ~~in half~~ as a result. What will be the impact on the threshold population?

$\Delta I < 0$

$aSI - bI < 0$

$(aS - b)I < 0$

$S < \frac{b}{a}$

p170

Threshold $= \frac{b}{a}$

$= \frac{\text{Pop.}}{(\text{recov. time})(\#\text{cont})(\text{ch inf})}$

A. Cut the threshold population in half.
B. Double the threshold population.
C. No changed in the threshold population.
D. Increase the threshold population by 2%.

In the basic SIR model, the parameter b is calculated as

A. 1/(time in S)
B. 1/(time in I)
C. 1/(time in R)
D. chance of infection/#contacts

$\frac{1}{4}$ cut den in half

→ $\frac{1}{2}$

recovery time = time spent getting to recovery group = time in I.

2. Consider the following two-species model:

$$\frac{\Delta x}{\Delta t} = 0.9x - 0.003x^2 - 0.006xy, \quad \frac{\Delta y}{\Delta t} = 0.8y - .004y^2 - 0.002xy$$

a) (5 points) Is either species logistic? How can you tell?

Both logistic. Both have squared terms

b) (5 points) Is this a predator prey model or competing species model? How can you tell?

Competing. Both have negative interaction (xy) terms

e) (5 points) If the starting populations are x = 100 and y = 100, will the species x and y increase or decrease in the near future?

Let $x = y = 100$ in $\Delta x = .9(100) - .003(100)^2 - .006(100)^2 = 90 - 30 - 60 = 0$

& $\Delta y = .8(100) - .004(100)^2 - .002(100)^2 = 80 - 40 - 20 = 20$

$\Delta x = 0 \rightarrow x$ stays the same

$\Delta y = 20 \rightarrow y$ increases

f) (10 points) Either

(i) Compare and contrast the two species in terms of their equations. How do they compare with regard to interactions? How do they compare with regard to other terms?

(ii) Or Sketch a phase plane for the two species.

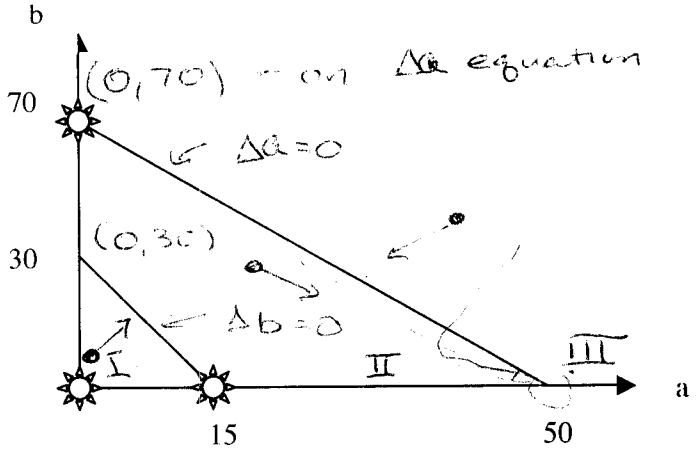
Suppose species a and b are modeled by the equations

$$\frac{\Delta a}{\Delta t} = 0.7a - 0.014a^2 - 0.01ab \quad = a(.7 - .014a - .01b)$$

$$\frac{\Delta b}{\Delta t} = 0.3b - 0.01b^2 - 0.02ab \quad = b(.3 - .01b - .02a)$$

a) Label the skew lines in the phase plane below as $\Delta a = 0$ or $\Delta b = 0$.
b) Add arrows to the regions of the diagram
c) Interpret the phase plane biologically.

$\Delta a = 0$ $\Delta b = 0$



Where ☼ denotes an equilibrium point.

(b)

I plot point (1,1)

$\Delta a = .7 - .014 - .01 > 0$

$\Delta b = .3 - .01 - .02 > 0$

II crosses $\Delta b = 0$

[illegible] changes from up to down

III crosses $\Delta a = 0$

[illegible] from right to left

(5)

NAME: ____

MATH 1470 Fall 2004 Tintera

TEST 3: Two-Species, Epidemic and Linear Programming Models. Covers Chapters 7-9

You may use calculators and one 8.5 by 11 inch page of notes. Please show all work on this test booklet. Partial credit is awarded only for work shown. Each problem is worth as indicated. Good luck!

.5% ch inf 25,000 12 contacts

1. A variation on the measles has broken out in a town of 25,000 people. In that town, people make 12 contacts per day and 5% of the contacts between an infected person and a susceptible person leads to the measles for the susceptible person. Since this is a new disease, no one is immune nor have they been vaccinated. It takes a week to recover from this type of measles.

7 days to recover

a) (10 points) Write the change model (Delta model) for this situation based on the SIR model.. ~~(I'll write the formulas for a and b on the board if you ask)~~.

$\frac{\Delta S}{\Delta t} = -.000024\, S I$

$\frac{\Delta I}{\Delta t} = .000024\, S I - \frac{1}{7} I$

$\frac{\Delta R}{\Delta t} = + \frac{1}{7} I$

a = # cont × ch inf = (12)(.05)/pop = 2.4E-5

b) (5 points) Early on in the outbreak, there are 12 people infected and none have recovered. How many will there be in each category (S, I, R) the next day?

$S_{next} = S_{today} + \Delta S$

$= (25{,}000 - 12 - 0) + -.000024(24988)(12)$

$= 24988 - 7.165 \approx 24981$

c) (5 points) Later, the number of infected has risen to 300. A reporter from a local newspaper asks you, the mayor of the town, if the number of infected will go up. Can you tell him?

$\Delta I > 0$

$aSI - bI > 0$

$aS - b > 0$

$S > \frac{b}{a}$

No. $\Delta I > 0$ depends only on S, not I. Need to know S before saying whether I will increase!

d) (5 points) For which values of S will I increase (ie, have $\Delta I > 0$) ?

Threshold Population