

11-29-05

①

## Games

### ① Rock-Paper-Scissors

2 players each choose Rock, Paper or Scissors  
winner chosen based on choices

### ② Prisoners' Dilemma

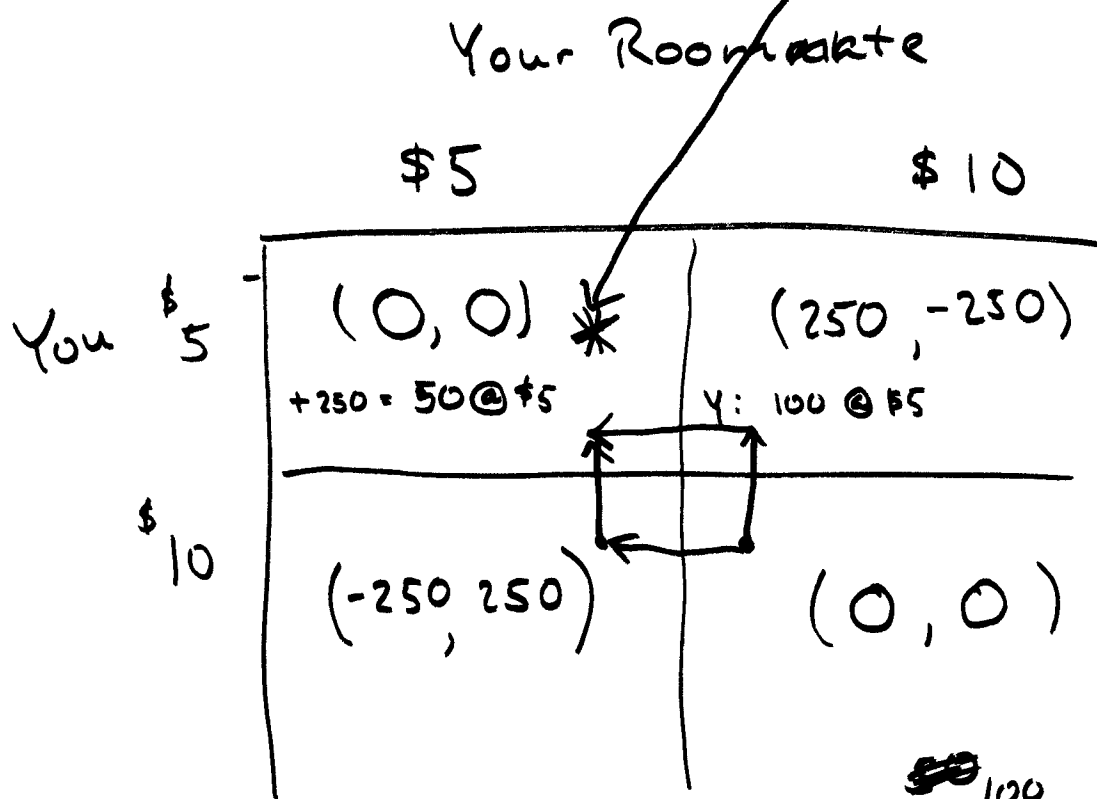
2 players each choose to Confess or not  
sentences are determined by choices

### ③ Selling Roses

2 players each choosing to sell roses  
for \$5 or \$10. Profit is determined  
by choice

Mathematical Games - 2 (or more) "players"  
each making a choice of "moves". Every  
combination of moves has a "payoff."

# Selling Roses Nash Equilibrium (2)



- Each spend \$250 to get roses
- At \$5, 100 roses will be sold
- At \$10, 50 roses will be sold

↔ means Roommate changed moves

↕ means You changed moves

\* Nash Equilibrium. Neither Player can improve payoff by ~~and~~ using a different strategy.

# Prisoner's Dilemma Roommate

		Confess	Not
You	Confess	$(-5, -5)$ ← 5 yr sentence each ↑	$(0, -10)$ Probation for You 10 yrs for Ro ↑
	Not	$(-10, 0)$ ←	$(-1, -1)$ 1 yr sentence on lighter crime,

PD

S R

$(-5, -5)$	$(0, -10)$
$(-10, 0)$	$(-1, -1)$

Not zero sum

0	250
<del><math>(0, 0)</math></del>	<del><math>(250, -250)</math></del>
-250	0
<del><math>(-250, 250)</math></del>	<del><math>(0, 0)</math></del>

Zero sum game  
 Sum of payoffs = 0  
 payoffs are equal but opposite

# Rock Paper Scissors - A Zero Sum Game

Roommate

(4)

You

	R	P	S	min
R	0	-1	1	-1
P	1	0	-1	-1
S	-1	1	0	-1
max	1	1	1	

No fixed strategy (choice of moves)

Can not be improved on by one player or another.

$$\begin{matrix} \text{maximin} = -1 \\ \text{minimax} = 1 \end{matrix} \quad \left. \vphantom{\begin{matrix} \text{maximin} \\ \text{minimax} \end{matrix}} \right\} \text{No saddle point.}$$

Nash Equilibrium IF  
 there is no saddle point, the  
 N.E. is in a mixed strategy.  
 (random)

		P <sub>2</sub>	
		x	y
P <sub>1</sub>	a	5	2
	b	3	7

2 = minimum payoff if P<sub>1</sub> chooses a

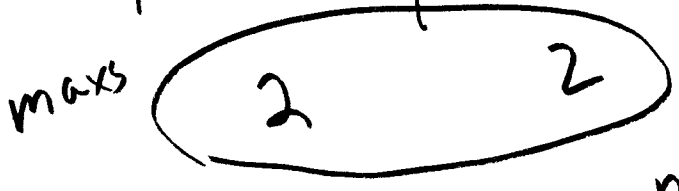
3 = min Payoff if P<sub>1</sub> chooses b

5 = max payoff if P<sub>2</sub> chooses x

7 = max payoff if P<sub>2</sub> chooses y

		P <sub>2</sub>	
		r	s
P <sub>1</sub>	c	2	-1
	d	-3	2

mins



Maxi min = -1  
mini max = 2

not = So No saddle point

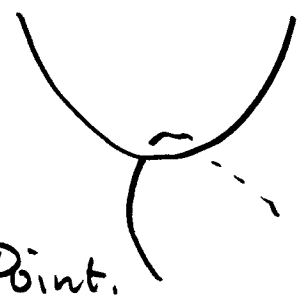
2

6

		M	N
1	e	2	3
	f	-1	4



2 = "saddle point"  
= Nash Equilibrium Point.



where minimax = maximin