

12-5-05

(1)

Grades 1000 points in Moodle

900 A

800 B

⋮

56	20%	Final	200	points
	24%	Labs	240	points
	10%	Reading	100	points (Open!)
	10%	HW	100	points.
	36%	Exams	360	points
			360	

$$\Delta S = -aSI$$

$$\Delta I = +aSI - bI$$

$$\Delta R = +bI$$

$$\Delta S + \Delta I + \Delta R = 0 + 0$$

1. Identify the following scenarios as having linear, exponential, or logistic models. Additionally, write appropriate equations for the models, specifying variables and values of parameters where possible.

- a) The number of computers affected by a virus starts at 10 and increases 25% each hour.

Exponential

start growth constant % increase

$$y = 10(1.25)^t$$

$y = \# \text{ computers}$ $t = \text{time (hours)}$

- b) The number of cars parking on campus grew at a rate of 10% per year for several years but will level off at the 2400 spaces available.

Logistic - b/c leveling off
* early growth

$$\frac{\Delta y}{\Delta t} = .10 y \left(1 - \frac{y}{2400} \right)$$

$y = \# \text{ cars}$ $t = \text{time (yrs)}$

carrying cap

- c) the average size of the crabs caught in Corpus Christi Bay was 11.8 ounces and has been decreasing by 0.2 ounces per year.

Linear - constant change in amount

$$y = -0.2t + 11.8$$

$y = \text{wt of crabs in oz}$ $t = \text{time (yrs)}$

2. The Corpus Christi Caller Times reported that mosquito repellent sales are up 550% at a local grocery store. Use this reported increase to express the relationship between last year's sales, $S(0)$, and this year's sales, $S(1)$, in two different ways.

Linear growth

$$\Delta S = S(1) - S(0) \quad \text{- difference}$$

absolute growth

exponential growth

$$\frac{\Delta S}{S} = \frac{S(1) - S(0)}{S(0)}$$

relative growth

3. Water is leaking from a swimming pool. After being left for some number of days, the drop of the level is measured in inches.

X (# days)	Y (inches)					
1	1					
5	2.1					
6.5	3					
8	5					

- a) Based on the data that appears above, would a linear model be appropriate for the relationship between time since the pool was full and the drop in level? (Your answer is less important than your justification of it.)

Look for constant (or nearly so)
 $\Delta Y / \Delta X$

- ~~X~~ b) Regardless of your answer to (a), what is the best linear model relating X and Y?

15 day vacation

- c) Using your model, predict the drop in level of the pool after a 15 day vacation. (If you can't do (b), use $Y = .75X + .8$.)

$$Y = .75X + .8$$

Y = drop in
water level

Let $X = 15$ days

$$Y = .75(15) + .8$$

⋮

4. On an island near New Zealand there are two species, Mutton Birds and Tiger Snakes. The chicks of the birds are the only food available for the snakes, and they can only eat the smaller chicks, within a week after the chicks are born. Otherwise, the chicks will become too large for the snakes to eat. The Lotka-Volterra Equations for the number of birds (B) and the number of snakes (S) are

$$\Delta B/\Delta t = .24 B - .01 BS - .008 B^2 \quad \text{and} \quad \Delta S/\Delta t = .001 BS - .1 S$$

a) Find the equilibrium points for the model.

$\Delta B = 0$ & $\Delta S = 0$

$\Delta B = 0$

$B = 0$ or $.24 - .01 S - .008 B = 0$

$S = 0$	$(0, 0)$ ✓	$S = 0$ $.24 - .008 B = 0$ $B = \frac{.24}{.008} = \frac{240}{8} = 30$
$\Delta S = 0$ $0 = .001 B - .1$ $B = \frac{.1}{.001} = 1000$	$B = 0$ $B = 1000$	$.24 - .01 S - .008(1000) = 0$ $-7.76 - .01 S = 0$ $S < 0$

Evolutionary biologists have found that the Tiger Snakes are adapting to their situation by increasing in size and being able to eat slightly older chicks. This has changed the equations to

$$\Delta B/\Delta t = .24 B - .024 BS - .008 B^2 \quad \text{and} \quad \Delta S/\Delta t = .002 BS - .1 S$$

b) Identify the coefficients that have changed and explain why they have been changed the way they have.

Interaction coefficients

In ΔB the interaction became more negative as birds lost to snakes
In ΔS , the coeffs increased. Snakes better

c) Bird lovers have suggested that, in order to be fair to the birds, the snakes should be removed from the island. What would happen to the population of birds in that case?

If $S = 0$

$$\frac{\Delta B}{\Delta t} = .24 B - .008 B^2$$

Logistic growth Expect
Birds to increase towards $L = \frac{a}{b}$

$$\text{or } L = \frac{.24}{.008} = \frac{240}{8} = 30$$

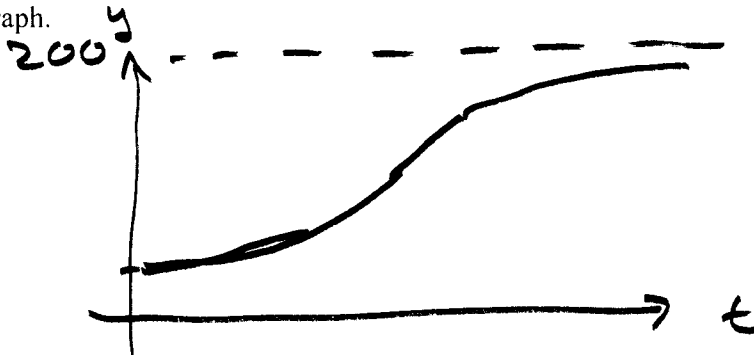
5. Rabbit print overalls, sold by S-Mart, are all the rage at day care centers across the country. When they were first introduced, the number sold was increasing at a rate of 2% per day. Marketing experts estimate that they will be able to sell 200 thousand of the overalls.

a) Write a logistic equation that models the number of overalls sold. Be explicit about the variables.

$$\frac{\Delta y}{\Delta t} = .02 y \left(1 - \frac{y}{200} \right)$$

$y = \# \text{ overalls (thousands)}$ $t = \text{time (days)}$

b) Sketch a graph of the number sold over time. Be sure to label the axes for your graph.



c) How many overalls would be sold in a day if 80 thousand had been sold up to that day?

sold in 1 day is Δy if $\Delta t = 1$

$$y = 80 \quad \Delta y = .02 (80) \left(1 - \frac{80}{200} \right)$$

d) For the following numbers of overalls sold, when is the number sold per day the greatest: After 10 thousand were sold, after 100 thousand were sold, or after 150 thousand were sold?

At 100 thousand = $\frac{1}{2}$ of carrying capacity.

8. In making cookies, Sarah Claus has available 300 sacks of flour, 90 sacks of chocolate, 40 jars of ginger, 3 bags of sprinkles, 10 hours of mixing time and 30 hours of oven time. She wants to make ginger snaps, apple crisps, chocolate cookies and bell cookies. The report below is from Ms. Claus' solution of how to make the greatest number of dozens of each kind of cookies.

Microsoft Excel 8.0 Sensitivity Report
 Report Created: 12/10/2001 11:13:16 AM

dozens

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
\$B\$2	Ginger Snaps	11.14361702	0	1	4	0.502994012
\$C\$2	Chocolate	60	0	1	1E+30	0.279255319
\$D\$2	Apple Crisps	144.2819149	0	1	1.012048193	0.8
\$E\$2	Bells	0	-0.558510638	1	0.558510638	1E+30

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
\$F\$4	Flour	67.7712766	0	300	1E+30	232.2287234
\$F\$5	Chocolate	49.1143617	0	90	1E+30	40.8856383
\$F\$6	Ginger	40	0.558510638	40	67.96812749	10.03592814
\$F\$7	Sprinkles	3	11.17021277	3	2.520577144	3
\$F\$8	Mixing	7.164228723	0	10	1E+30	2.835771277
\$F\$9	Oven	30	5.319148936	30	8.38	21.7

a) How many dozens of each type of cookie and cookies in all should Ms. Claus make?

See # dozens above

e) Should Ms. Claus buy a mixer or an oven?

Shadow Price = 0
 Extra 5+ dozen cookies for each hour of oven time.

f) If Ms. Claus had more chocolate, would she be able to make more cookies? If so, how many more? If not, why not?

No, shadow price = 0

↓ Yes, shadow price > 0. Get 65 jars of ginger.

g) If Ms. Claus had more ginger, would she be able to make more cookies? If so, how many more? If not, why not?