NAME: Key

MATH 1470 Fall 2005 Tintera

TEST 3: Two-Species and Epidemic Models. Covers Chapters 7-8

You may use calculators and one 8.5 by 11 inch page of notes. Please show all work on this test booklet. Partial credit is awarded only for work shown. Each problem is worth as indicated. Good luck!

For Questions 1 through 5, circle the best response.

1. Species r and s are represented in a plane with r on the horizontal axis and s on the vertical axis. In a portion of the plane, $\Delta r < 0$ and $\Delta s > 0$. In that portion of the plane

A. r increases and s increases.
B. r increases and s decreases.
C. r decreases and s increases.
D. r decreases and s decreases.

2. What is the name of the graph with the two species on the two axes?

A. Poincare Plane
B. Equilibrium Plane
C. Phase Plane
D. Jet Plane

3. The basic SIR model for measles is modified to allow for the possibility that people lose their immunity to the measles after a while. Which is true about the change in the diagram for the model?

A. A new arrow is added but no new groups
B. A new group is added to the model but no arrows
C. Both a new arrow and a new group are added
D. The diagram is unchanged.

4. A town experiencing a measles epidemic suddenly doubles in population with hurricane refugees. The new residents have the same social habits (# contacts, chance infection per contact) as the original towns people. What will be the impact on the threshold population?

A. Cut the threshold population in half.
B. No changed in the threshold population.
C. Double the threshold population.
D. Increase the threshold population by 2%.

5. In the basic SIR model, which of the following is true?

A. $\Delta S = -\Delta I$
B. $\Delta S = \Delta R$
C. $\Delta R < 0$ at some time.
D. $\Delta S + \Delta I + \Delta R = 0$

6. In January 2005 there was considerable concern about the possibility of an influenza outbreak, especially because of a shortage of vaccine. An article "Influenza activity is normal" by Leanne Libby appeared in the Corpus Christi Caller-Times on January 22, 2005. For the purpose of this question, assume that the influenza in Corpus Christi can be modeled by the basic SIR model from the text.

a) (5 points) The subtitle of that article was '11 case per week in Corpus Christi.' Circle the mathematical statement that best represents that subtitle. In all cases, assume $\Delta T = 1$ week.

A. $\Delta S = -11$
B. $\Delta I = -11$
C. $b = 1/11$
D. $\Delta R = 11$
E. $\Delta I = 11$

b) (3 points each) The article included four tips from the Centers for Disease Control and Prevention that were supposed to help prevent flu. For each of the tips, identify the factor in the formula for 'a' that is directly changed by having people follow that tip: $a = \frac{\text{Chance infection} \times \#\text{contacts}}{\text{tot pop}}$

A. Stay away from people who are sick.

Factor: # contacts

B. Wash hands often with soap and water, especially after you cough or sneeze. If you are not near water, use an alcohol-based hand cleaner.

Factor: ch inf , ~~# contacts~~

C. Cover nose and mouth with a tissue when you cough or sneeze; throw tissues away after use.

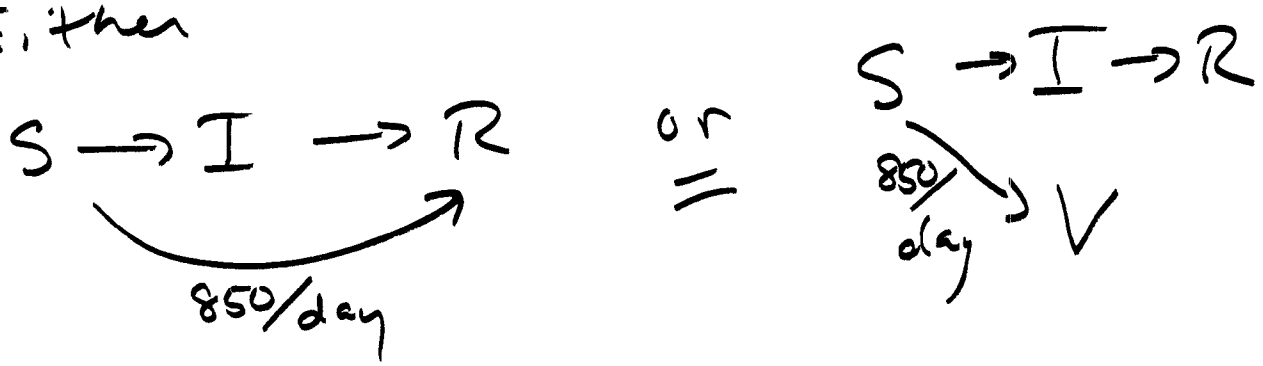
Factor: ch infection, # contacts

D. Try not to touch your eyes, nose or mouth. Germs often spread this way.

Factor: ch infection

c) (8 points) Suppose a program of vaccinating 850 people per day for 100 days is implemented in Corpus Christi. Show a modification of the basic SIR diagram that represents this vaccination program. **You do not need to write the Delta equations for the model.**

Either



7. Consider the following two-species model:

$$\frac{\Delta x}{\Delta t} = 0.004xy - 0.6x, \qquad \frac{\Delta y}{\Delta t} = 0.75y - 0.0015xy - .025y^2$$

+ interaction − interaction

a) (6 points) Is either species logistic? How can you tell?

y is logistic. It has a squared term
x isn't " " " no " "

b) (6 points) Is this a predator prey model or competing species model? How can you tell?

Predator-Prey: x is the predator with a positive interaction coefficient. y is the prey (negative coefficient)

c) (6 points) If the starting populations are x = 100 and y = 100, will the species x and y increase or decrease in the near future?

$\Delta X = .004(100)(100) - .6(100) = 40 - 60 < 0$

$\Delta y = .75(100) - .0015(100)(100) - .025(100)^2 = 75 - 15 - 250 < 0$

They both decrease!

d) (7 points) Find the equilibrium points for this model.

Solve $\Delta X = 0$ & $\Delta y = 0$ simultaneously

(0,0) & (0,30)

$\Delta y = 0$

$y=0$	or $.75 - .0015x - .025y$
(0,0)	$x=0 \Rightarrow .75 - .025y = 0$, $y = \frac{.75}{.025} = 30$, (0,30)

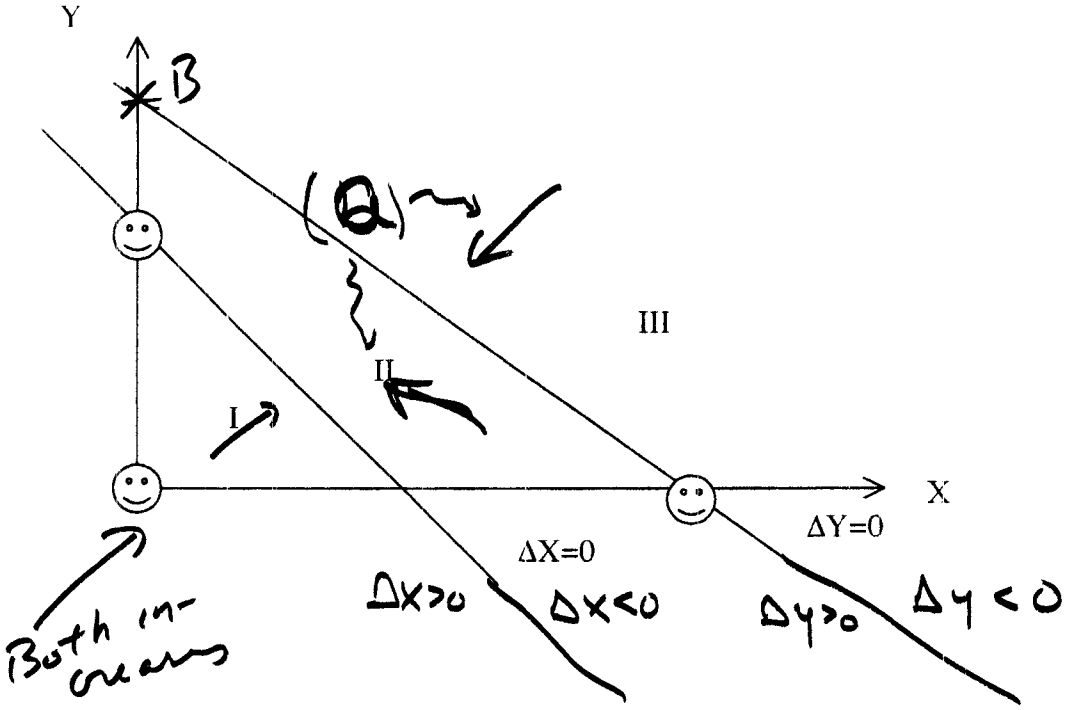
$\Delta X = 0$ or

$x=0$	$y=0$	$y = 150 \Rightarrow x < 0$
$.004y - .6 = 0$ ($y = \frac{.6}{.004} = 150$)	$y = 150$ can't happen	

8. Below is the phase plane for a pair of species X and Y. In Region I, it is known that $\Delta X > 0$ and $\Delta Y > 0$. ☺ is an equilibrium point.

a) (5 points each) Draw arrows to show the direction of change of the points in Region II and Region III.

b) (15 points) Interpret the phase plane biologically.



(b) In Regions I, III the points head towards Region II.

In Region II the points head towards B

In biological terms, this would mean extinction for X and a constant pop for Y.