Math 2305, Key, Test 1

1.

A	B	$A \vee B$	$A \wedge B$	$A \to B$	$(A \vee B)' \to (A \to B)$
T	T	T	T	T	T
T	F	T	F	F	T
F	T	T	F	T	T
F	F	F	F	T	T

2 pts. $\quad 4 \times 4 = 16$ @ $\frac{1}{2}$ pt each $= 8$ pts total.

2. Modus Ponens: $P \wedge (P \to Q) \to Q$.

P	Q	$P \to Q$	$P \wedge (P \to Q)$	$P \wedge (P \to Q) \to Q$
T	T	T	T	T
T	F	F	F	T
T	T	T	T	T
F	F	F	F	T

$2\frac{1}{2}$ pts each

So $P \wedge (P \to Q) \to Q$ is a tautology.

3. (1) Let P represent "the number ~~2312~~ 231231 is divisible by 7"
(1) and Q " " "the Gulf of Mexico is filled w/ Jello."
(2) ① The ~~~~ statement representing it is $P \vee Q$
(3) ② The negation is $(P \vee Q)'$, or $P' \wedge Q'$
(3) ③ The negation of the original is: "The number 231231 is not divisible by 7 and the Gulf of Mexico is not filled with Jello."

(2) 4. ① Let x & y be the two odd integers
(2) ② So $x = 2n+1$ & $y = 2m+1$ for some integers n, m.
(4) ③ Then $x \cdot y = (2n+1)(2m+1) = 4nm + 2n + 2m + 1$

$= 2(2nm + n + m) + 1$

(2) ④ Since $2nm + n + m$ is an integer $x \cdot y$ is odd.

5. All fish have scales

(2) Let $F(x)$ represent x is a fish
& $S(x)$ " x has scales.

(3) ① $(\forall x)\ F(x) \rightarrow S(x)$

(3) ② $[(\forall x)\ F(x) \rightarrow S(x)]' = (\exists x)\ (F(x)' \vee S(x))'$
$= (\exists x)[F(x) \wedge S'(x)]$.

(2) ③ "There are fish that don't have scales."

6. 1. hypothesis (1)
2. hypothesis. (1)
3. modus tollens, 1, 2. (2)
4. NOT JUSTIFIED. (1)
5. 3 dn (double negation) (2)
6. hypothesis (1)
7. 5, 6 modus ponens. (2).

(3) 7. a) If "x & y are odd integers" then "$x \cdot y$ is odd"

(4) b) If "$x \cdot y$ is even" then "x & y are not both odd"

(3) c) Hypotheses
- x is odd
- y is odd
- $x \cdot y$ is even.

8. $P(n)$: $8^n - 3^n$ is divisible by 5.

(4) Basis : $P(1)$: $8^1 - 3^1 = 5 = 5 \cdot 1$ is divisible by 5.

(2) Assume $P(k)$: $8^k - 3^k = 5 \cdot j$ for some integer j.

(4) Prove $P(k+1)$: $8^{k+1} - 3^{k+1} = 5 \cdot k$ for some integer k.

For this proof : $8^{k+1} - 3^{k+1} = 8 \cdot 8^k - 3 \cdot 3^k$
$= 8 \cdot (3^k + 5j) - 3^{k+1}$ (By induction hypothesis) $= 5(j \cdot 8) + 8 \cdot 3^k - 3 \cdot 3^k$
$= 5 \cdot j \cdot 8 + 3^k(8-3) = 5(8j + 3^k)$. So $8^{k+1} - 3^{k+1}$ is divisible by k.

9. ~~9.~~ Mystery(2) = Mystery(1) + 3 = 4
Mystery(3) = Mystery(2) + 3 = 4 + 3 = 7.
Mystery(4) = Mystery(3) + 3 = 7 + 3 = 10
Mystery(5) = Mystery(4) + 3 = 10 + 3 = 13
Mystery(6) = Mystery(5) + 3 = 13 + 3 = 16.

2 pts each

10. a) Let $B(n)$ = pop of Brazil n years after 2000. in millions

So 1. $B(0) = 173$
2. $B(n) = 1.01 \times B(n-1)$ for $n \geq 1$.

4 ~~5~~ pts

b)

n	0	1	2	3	4	5.
$B(n)$	173	174.73	176.477	178.241	180.023	181.823

n	6	7	8	9	10
$B(n)$	~~181.641~~	~~183.641~~	~~185.478~~	~~187.332~~	~~189.206~~
	183.641	185.478	187.332	189.206	191.098

6 ~~8~~ pts

(Year 2010 is 10 yrs after yr 2000).

There will be (assuming 1% growth rate holds) 191 ~~189~~ million, 98 ~~206~~ thousand people in Brazil in 2010.

11. (2) Conjecture Mystery$(n) = 3(n-1) + 1$. or $= 3n - 2$
(2) Basis Mystery$(1) = 1$, $3(1-1) + 1 = 0 + 1 = 1$.
(2) Assume $P(K)$: Mystery$(k) = 3(k-1) + 1$
(2) Prove $P(K+1)$: Mystery$(K+1) = 3K + 1$
(2). proof: Mystery$(K+1)$ = Mystery$(K-1) + 3$
$= 3(K-1) + 1 + 3$
$= 3K - 3 + 1 + 3$
$= 3K + 1$ ✓.