MATH 2305 Spring 2003 Test 2 Name: Key

Part 1. You may not use any notes, books or calculators for this part of the test. You will receive Part 2 after you turn in Part 1. Be sure to leave plenty of time to finish Part 2. No partial credit without work shown.

1. Let A = { 1, 2, 3, 4, 5} and B = {3, 4, 5, 6, 7}. Find the following

a) $A \cup B$ $= \{1, 2, 3, 4, 5, 6, 7\}$

b) $A \cap B$ $= \{3, 4, 5\}$

c) $A - B$ $= \{1, 2\}$

d) $|A \times B|$ (A x B is the Cartesian product of A and B) $|A \times B| = 5 \cdot 5 = 25$

e) $|\mathcal{P}(A)|$ ($\mathcal{P}(A)$ is the power set of A) $|\mathcal{P}(A)| = 2^{|A|} = 2^5 = 25$

2. A restaurant has 3 different salads, 10 vegetables, 4 entrees, 6 types of pie and 4 types of cake. Find the following

a) The number of different possible desserts (one piece of pie or one piece of cake).

$10 = 6 + 4$

b) The number of different possible dinners consisting of a salad, a vegetable, an entrée and a dessert.

$1200 = 3 \cdot 10 \cdot 4 \cdot 10$

c) The number of ways to select 2 vegetables from the 10 offered.

$C(10,2) = \frac{10 \cdot 9 \cdot \cancel{8!}}{\cancel{8!}\, 2!} = \frac{90}{2} = 45$

3. A group of 15 individuals is asked if they liked cola or lemonade. There were 8 who said they liked cola and 4 who liked both cola and lemonade. How many people like lemonade?

Let A = set of people who like cola & B = set of people who like lemonade

$15 = 8 + X - 4$

$|A \cup B| = |A| + |B| - |A \cap B|$

$X = |B| = 11$

So 11 people like lemonade

4. Prove the statement: For sets A and B, if $A \subseteq B$, then $\mathcal{P}(A) \subseteq \mathcal{P}(B)$ ($\mathcal{P}(A)$ is the power set of A.)

Choose $S \in \mathcal{P}(A)$. So $S \subseteq A$. Since $A \subseteq B$

$S \subseteq B$. This means $S \in \mathcal{P}(B)$. Since every

element in $\mathcal{P}(A)$ is in $\mathcal{P}(B)$, $\mathcal{P}(A) \subseteq \mathcal{P}(B)$.

0!

$$S \in \mathcal{P}(A) \Rightarrow S \subseteq A \subseteq B$$
$$\Rightarrow S \subseteq B$$
$$\Rightarrow S \in \mathcal{P}(B)$$

So $\mathcal{P}(A) \subseteq \mathcal{P}(B)$.

5. There are 5 blue marbles, 7 red marbles, 3 grey stones and 5 red dice in a bag. A single object is drawn from the bag. Find:

$5+7+3+5=20$

a) The probability that the single object is a blue marble.

$$\frac{5}{20} = \frac{1}{4}$$

b) The probability that the object is red.

$$\frac{7+5}{20} = \frac{12}{20} = \frac{3}{5}$$

c) The probability that object is not red.

$$\frac{5+3}{20} = \frac{8}{20} = \frac{2}{5}$$

d) The probability that the object is grey and a marble.

$$0$$

e) The probability that the object is a marble given that the object is red.

$$\frac{\#\text{marbles}}{\#\text{red objs.}} \quad \frac{7}{12}$$

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Part 2. You may not use any books for this part of the test. You may only use a single sheet of notes and a calculator. Be sure to answer all questions. No partial credit if work is not shown. Good luck!

6. A convoy contains two trucks (one for fuel, another for other supplies), four tanks and two hummers and is proceeding in a single file. (It doesn't really matter what a hummer is).

a) In how many ways can the vehicles in this convoy be arranged?

8 vehicles $\Rightarrow P(8,8) = 8! = \#$ arrangements

b) In how many ways can the convoy be arranged if the first vehicle is required to be a tank and the last vehicle must be a hummer?

First find 1st vehicle then last vehicle, then middle vehicles

$\#$ ways $= 4 \cdot 2 \cdot (6!)$

c) In how many ways can the convoy be arranged regardless of the first and last vehicles if the tanks cannot be distinguished from one another and the hummers cannot be distinguished from one another.

$$\frac{8!}{4!\cdot 2!} = \frac{\overset{4}{\cancel{8}}\cdot 7\cdot 6\cdot 5\cdot \cancel{4!}}{\cancel{2}\cdot \cancel{4!}} = 840$$

7. Let A be the event that a students is smart and B be the event that the student is rich. Express the following events in terms of A and B.

a) The student is smart and rich. $A \cap B$

b) The student is smart but not rich. $A \cap B'$

c) The student is not rich. B'

d) The student is either not smart or is rich. $A' \cup B$

e) The student is rich or smart. $B \cup A$

8. Suppose a student drawn at random from the university role. The probability that a student is smart is .74. The probability that a student is rich is .18. The probability that a student is both smart and rich is .10. Find the probability that

Let S = event that student is smart, R = student is rich.

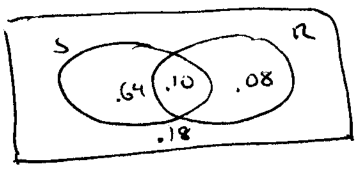
a) The student is smart and rich. $P(S \cap R) = .10$

b) The student is smart but not rich. $= .64$

c) The student is not rich. $P(R') = 1 - P(R) = 1 - .18 = .82$

d) The student is either not smart or is rich. $= .08 + .18 \cancel{+ .10} + .10 = \cancel{.26}\ .36$

e) The student is rich or smart. $P(R \cup S) = .74 + .18 - .10 = .82$



9. A carton of one dozen eggs has two that are bad. A cook will keep choosing eggs until a good one is found.

a) What is the greatest number of eggs that the cook might choose before getting a good one.

Choosing the two bad eggs & then finally a good one would require choosing 3 eggs.

b) Find the sample space for the egg choosing. Also, find the probability for each outcome in the sample space.

The cook must choose at most 3 eggs. So she could choose ~~0~~, 1, 2, or 3 eggs.

1st one good; 1st bad, 2nd good

x	1	2	3
$p(x)$	$\frac{10}{12}$	$\frac{2}{12} \cdot \frac{10}{11}$	$\frac{2}{12} \cdot \frac{1}{11} \cdot \frac{10}{10}$

10. The number of people that goes to an outdoor concert depends on the weather. If the weather is sunny, 2500 people will go. If the weather is overcast, 1800 people will attend. If the weather is rainy, only 500 people will go. The weatherman says there is a 20% chance of sunny weather and a 25% chance of rain. Otherwise it will be overcast. What is the expected number of people that will go to the concert? (The answer will be a single number).

Let x = weather, $f(x)$ = # people that attend & $p(x)$ = probability of the weather

x	sunny	overcast	rainy
$f(x)$	2500	1800	500
$p(x)$	.20	.55	.25

Expected # people = $\sum f(x)\cdot p(x) = (2500)(.20) + (1800)(.55) + 500(.25)$

$= 500 + 990 + 125$

$= 1615$

11. Bonus: The new Texas Lotto will have 44 numbers. The first 5 numbers will be chosen without replacement from the same set of 44 numbers. A 'power ball,' the 6th number, will be chosen from a different set of 44 numbers. Find the number of possible distinct tickets that could be sold under this new scheme.

To count tickets, first find the # of regular numbers & then the power ball. The number of ways to do this is

$$\underset{①}{C(44,5)} \cdot \underset{②}{C(44,1)}$$

$$= \frac{44!}{5!\,39!} \cdot \frac{44!}{43!\cdot 1!} = \frac{44\cdot 43\cdot \overset{14}{\cancel{42}}\cdot 41\cdot \cancel{40}\cdot \cancel{39!}}{\cancel{5}\cdot\cancel{4}\cdot\cancel{3}\cdot\cancel{2}\cdot 1\cdot \cancel{39!}} \cdot \frac{44\cdot \cancel{43!}}{\cancel{43!}\cdot 1}$$