4. Consider the following two-species model:

$$\frac{\Delta x}{\Delta t} = 0.6x - 0.005x^2 - 0.006xy, \qquad \frac{\Delta y}{\Delta t} = -0.05y + 0.001xy$$

a) Is either species logistic? How can you tell?

X is logistic. Because of the x^2 term. (Δy doesn't have a y^2 term)

b) Is this a predator prey model or competing species model? How can you tell?

Predator-Prey. The Δx interaction term is negative, making x the prey. Species y is the predator because of the positive interaction term.

c) Find the equilibrium points for this model.

d) The Phase Plane for the model is sketched below. Label the equilibrium points on the model.

I Solve $\Delta x = 0$ $\quad x(.6 - .005x - .006y) = 0$

$x = 0$ or $.6 - .005x - .006y = 0$

y-axis $\quad .6 = .005x + .006y$

points $(0, 100), (120, 0)$

II Solve $\Delta y = 0$ $\quad y(-.05 + .001x) = 0$

$y = 0$ or $-.05 + .001x = 0$

III Draw lines $\quad .001x = .05$

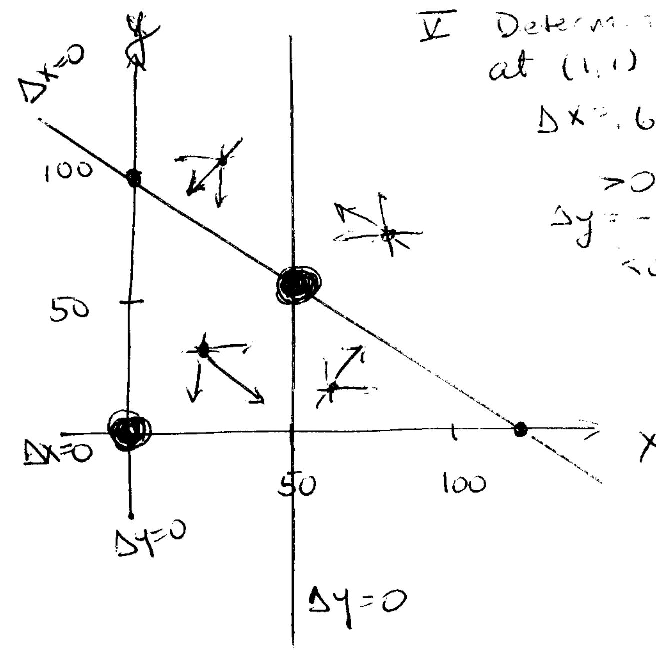
$x = 50$

IV Eq pts where $\Delta x = \Delta y = 0$. $\quad$ a vertical line

V Determine sign at (1,1)

$\Delta x = .6 - .005 - .006 > 0$

$\Delta y = -.05 + .001 < 0$



Assume a starting population of x = 10 and y = 110.

e) What is likely to happen in the near future for the species x and y?

If $x = 10$, $y = 110$, $\Delta x = -1.1$, $\Delta y = -4.4$. So both species will decrease in number. Or look at phase plane.

f) What is the long term biological interpretation of the model?

The populations of the species will alternately increase and decrease. In the phase plane, the trajectory for the pair (x, y) will spiral around the equilibrium point (50, 58).

3. An incomplete S-I-R model for a measles epidemic is as follows:

$$\frac{\Delta S}{\Delta t} = -0.0003SI, \quad \frac{\Delta R}{\Delta t} = 0.125I$$

$S \to I \to R$

a) Write the change equation for I to complete the model

$$\frac{\Delta I}{\Delta t} = +0.003SI - 0.125I$$

b) The spreadsheet below implements this model for village with a population of 10,000 people.

	A	B	C	D
1	a	0.0003		
2	b	0.125		
3				
4	Day	S	I	R
5	0	9998	2	0
6	1			
7	2			

What formula goes in the cell C6? $I_2 = I_1 + \Delta I = C5+(\$B1 * B5 * C5 - \$B2 * C5)$

Below are some scenarios that could change the model. For each scenario, indicate which of the following would change and how they would change (increase/decrease): a, b, S in B5, I in C5, R in D5.

c) Half the village left to go to market on Day 0, and those who were infected stayed home.

pop cut in half leads to doubling of a

S- ~~5000~~ 4998

I- 2

d) A circus comes to town on Day 0, and half of the members of the troupe of 40 have the measles.

pop incr → a decreases

S = 10018 = 9998 + 20

I = 2+20 = 22

e) The village has a festival where there is more feasting, dancing and merriment than usual.

S, I, R same

contacts up → a increasing

Hints for graphing phase planes

I : Sketching lines: $\Delta X = 0$.

① First Factor ΔX eg. $\Delta X = .6X - .005X^2 - .006xy$

has a common facter of x in each term

So $\Delta X = X(.6 - .005X - .006y) = 0$.

② Set each factor $= 0$, put in standard form

$X = 0$ or $.6 - .005X - .006y = 0$

$.6 = .005X + .006y$

(Note some facters may be vertical lines/horizontal lines eg. $ax - b = 0$, $cy - d = 0$)

③ ~~Plot~~ Find intercepts for non-horizontal/vertical lines to get points.

If $X = 0$: $.6 = \cancel{(.005)(0)}^{\,0} + (.006)y$, $100 = \frac{.6}{.006} = y$ $\longrightarrow (0, 100)$

If $y = 0$: $.6 = (.005)x + \cancel{(.006)(0)}^{\,0}$, $X = \frac{.6}{.005} = 120$. $\longrightarrow (120, 0)$

④ Then plot lines by connecting above points.

II Determining direction of trajectory

1. Pick a point for which it easy to calculate Δx, Δy. Try $(x,y)=(1,1)$

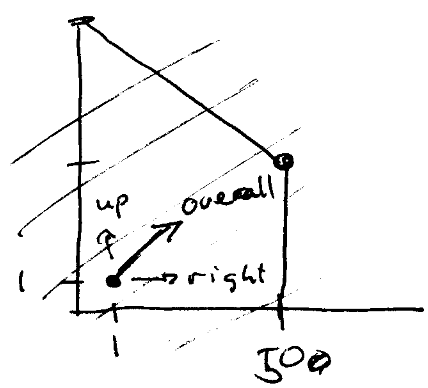
e.g.
$$\Delta x = (.6)(1) - (.005)(1)^2 - (.006)(1)(1)$$
$$\approx .6 > 0$$

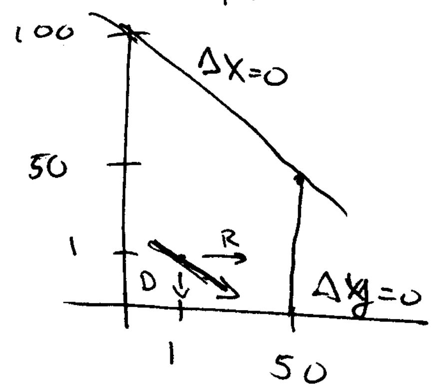
$$\Delta y = -.05(1) + (.001)(1)(1)$$
$$= -.05 + .001$$
$$\approx -.005 < 0.$$

So x will increase (move to right) and y will decrease (move down)

i.e.

2. Determine direction of (x,y) in region





(3) Determine direction of (x,y) in 2nd region across $\Delta x=0$ line or $\Delta y=0$ line

- Crossing $\Delta x=0$ line
  x direction changes
- Crossing $\Delta y=0$ line
  y direction changes