NAME: Key

MATH 1470 Spring 2004 Tintera

TEST 2: Malthus, Demographic Transition and Logistic Models. Covers Chapters 4-6

You may use calculators and one 8.5 by 11 inch page of handwritten notes. Please show all work on this test booklet. Partial credit is awarded only for work shown. Each problem is worth as indicated. Good luck!

1. The following data is about the population and banana production in Costa Rica: The data may be familiar, but the problems are new!

| Year | Population | Bananas | Increase in population | % increase in population | $\frac{\Delta B}{\Delta t}$ | 3 yr centered avg |
|---|---|---|---|---|---|---|
| 1991 | 3144 | 1720 | | | | |
| 1992 | 3245 | 1920 | 101 | 3.16% | 200 | |
| 1993 | 3349 | 1500 | 104 | 3.15% | 420 | 80 |
| 1994 | 3452 | 2000 | 103 | 3.03% | 500 | 120 |
| 1995 | 3554 | 2300 | 102 | 2.91% | 300 | 300 |
| 1996 | 3652 | 2400 | 98 | 2.72% | 1000 | 100 |
| 1997 | 3748 | 2300 | 96 | 2.59% | -100 | 60 |
| 1998 | 3841 | 2500 | 93 | 2.45% | 200 | |

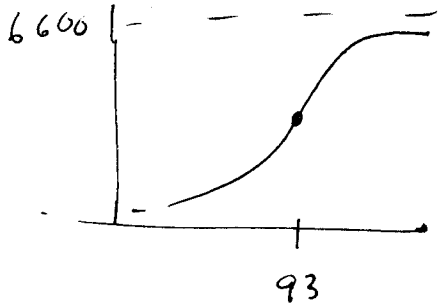
a) Does the production of bananas in Costa Rica match what Malthus said about food production in general? Be clear about what he said, you see and your conclusion. (1) (2) (3)

(1) M said it grows arithmetically (linearly)

15 (2) Since $\Delta B/\Delta t$ is (is not) constant, the Banana production is (is not) linear.

(3) So Banana production in Costa Rica does (does not) agree w M's theories about Food Prod

b) From what we have seen of human populations it is reasonable that the population of Costa Rica would grow logistically. Write down a logistic model and sketch its curve for the population of Costa Rica, labeling 1993 on the time axis. Be as detailed as possible. (Hint: Pay attention to the increase and % increase of the population and how they determine the parameters, r and L in the logistic equation.)

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Since pop'n increases the most in 1993, the pop'n at that time must be half the carrying capacity. So the carrying cap. is $\approx 2 \cdot 3300 = 6600$

$$\frac{\Delta y}{\Delta t} = 0.0316\, y\left(1 - \frac{y}{6600}\right)$$

$y$ = # Costa Ricans

$t$ = time (yrs)

2. The number of library cards in Corpus Christi seems to be governed by a the logistic model:

$rx - \frac{r}{L}x^2 = rx\left(1 - \frac{x}{L}\right)$ $\quad = \quad \frac{\Delta x}{\Delta t} = 0.125x - 0.0000045x^2$

a) Find the maximum number of library cards in Corpus Christi predicted by the model.

~~r=0.125~~

By matching the general equation with the one for Library cards. $r = 0.125$, $\frac{r}{L} = 0.0000045$

So $L = \frac{0.125}{0.0000045} = 27,778$

b) If there were 10,000 library cards in Corpus Christ one year, how many would there be the next year?

W/ $\Delta t = 1$ $\quad \Delta x = .125(10,000) - .0000045(10,000)(10,000)$

$= 1250 - 450$

$= 800$

So next year there will be 10,800

3. For years, the number of people playing golf rose at about 3% but seems to be leveling off at 25 million.

a) What type of model is appropriate for this situation? Explain.

$\frac{\Delta g}{\Delta t} = .03\, g\left(1 - \frac{g}{25}\right)$

$t = $ time

$g = $ # golfers in millions

b) Write the equation for the model you chose. Be sure to explicitly define the variables used.

A Logistic model is appropriate

Since there is early exponential growth but the quantity levels off.

4. Below is the population and food supply for the country of Thailand for the years given.

| Year | Population (in 1000's) | Food supply |
|---|---|---|
| 1995 | 17,743 | 99.0 |
| 1996 | 17,942 | 111.2 |
| 1997 | 18,138 | 121.6 |
| 1998 | 18,329 | 119.7 |
| 1999 | 18,517 | 123.8 |

In the problems below, please show enough work so that I know how you got your answers.

(*a*) Find the food supply per capita in 1997.

$\frac{\text{food}}{\text{pop}} = \frac{121.6}{18,138} = 0.00670$

(*b*) Find the indexed population for 1997, using 1995 as your base year.

$100 \times \frac{97\ \text{pop}}{95\ \text{pop}} = 100 \times \frac{18138}{17743} = 102.23$

(c) By what percent did the population increase from 1995 to 1997?

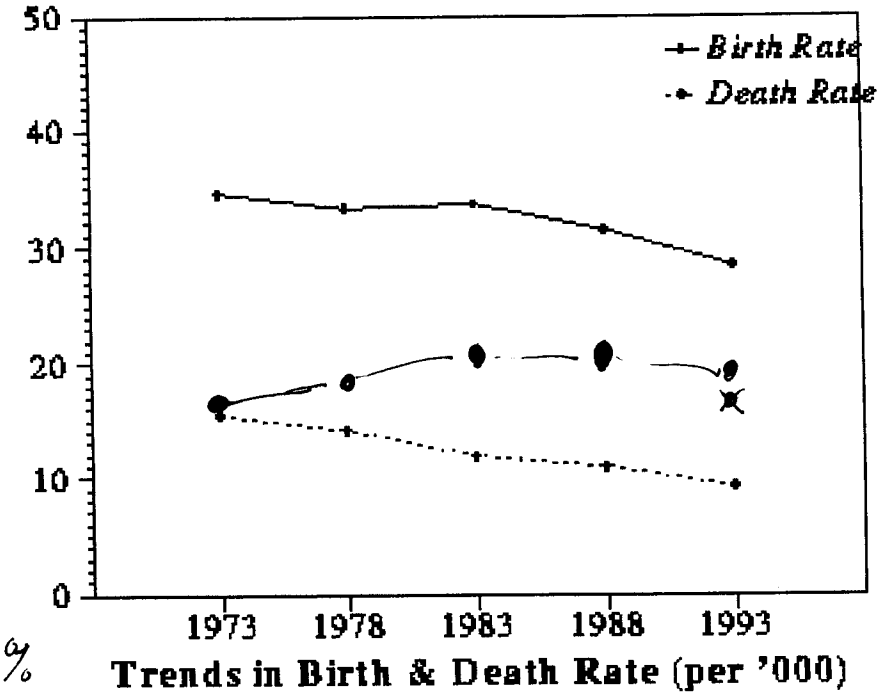
Above answer − 1 expressed as a %

2.23%

(*d*) Find the three-year trailing moving average food supply for 1999.

$\frac{123.8 + 119.7 + 121.6}{3} = 121.7$

5. (4 points each). The chart to the right gives the birth and death rates (as #'s per thousand population) for India from 1973 to 1993.[1]




88 | 34-12
83 | 35-13
78 | 21-16
135-17

a) From the chart, determine the birth, death and growth rates for India in 1993. Express them as a percentage of the population of India.

-Birth Rate: $\frac{30}{1000} = \frac{3}{100} = 3\%$

-Death Rate: $\frac{10}{1000} = \frac{1}{100} = 1\%$

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Growth Rate: $3\% - 1\% = 2\%$

b) On the chart above, sketch the growth rate of India over the entire time span of 1973 to 1993.

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c) Use the chart above to determine the stage of demographic transition is in as of 1993. Your answer should be specific enough that I know you understand what are the characteristics of that stage and why you chose your answer.

Either The growth rate has risen and leveled off. A high growth rate is characteristic of Level II

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or

The ~~gro~~ birth rate is falling after 1983 while the death rate ~~is~~ ~~cont~~ started falling earlier. So the country is past Stage II when the death rate is falling but the birth rate is not.

[1] http://www.indolink.com/Consulate/iebo/1_2_3bir.htm