MATH 4306—Modern Algebra Mid-Term Exam March 14, 2002

Name:

Instructions:

1. Each numbered problem has two parts, A. and B.
2. Part A of each numbered problem is worth a possible 10 points.
3. Part B of each numbered problem is worth a possible 30 points.
4. Do a total of 4 part A's and 2 part B's for a possible 100 points total.
5. Circle the 4 part A's and 2 part B's on the chart below to indicate which you want graded.
6. Only those circled parts will be graded.
7. Show all work for full credit.
8. You may use your notes and textbook during the test.
9. You may not use information from any other person during the test.

Question	Parts		Scores
1.	A	B	
2.	A	B	
3.	A	B	
4.	A	B	
5.	A	B	
6.	A	B	
Total			

10. You may take a copy of this test with you to work on for additional credit under the following rules:
    A) You can only earn additional credit for the problems circled above.
    B) Additional work should include a complete solution of the problem.
    C) All extra work done is due at the start of class on Tuesday, March 26, 2002.
    D) All extra work must be done without consulting any other person (in the class or otherwise) or text or materials.
    E) Your final score for each problem will be the average of your original score and the score on your additional work..

2.A. Let G = { (a,b) : a in Z_2 and b in Z_4} Define a product on G by
(a,b) ∘ (c,d) = ((a+c) mod 2, (b+d) mod 4). Find

i) The product (1,3) ∘ (0,2)
ii) The product (0,3) ∘ (1,0)
iii) The identity of the operation ∘ (if there is one)
iv) The inverse (if there is one) of the element (1,1) of G.

2.B. Suppose (G_1, o_1) and (G_2, o_2) are groups. Let G = { (a,b) : a in G_1, b in G_2 } and define an operation # on G by (a,b) # (c,d) = (a o_1 c, b o_2 d). Show that G is a group with the operation #.

1.A. For the permutations (in cycle notation) p = (2 1 4) and q = (3 6) show that $(p \circ q)^n = p^n \circ q^n$ for n = 1,2,3,4,5,6.

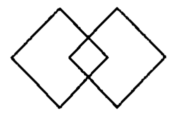
1.B. Show that for ANY two disjoint permutations p and q and ANY integer n, $(p \circ q)^n = p^n \circ q^n$.

3.A. In which of the following binary systems does the equation 2 x = 6 have a unique solution?

i. G = Z_{11} – {0}, with operation multiplication mod 11.
ii. G = Z_{12} – {0} with operation multiplication mod 12.
iii. G = Z_{15} – {0} with operation multiplication mod 15.

3.B. Suppose a,b,c,d,x are members of a group G with operation *. Solve the following equation for x in G, giving careful justifications for each step in the solution: $a*b^2*c = a*b*x*c*d^{-1}$.

4.A. For the given figure, find all possible symmetries (mappings of the figure to itself) and create a multiplication table for the operation of composition of those symmetries. (Yes, the figure contains both 'diamonds').



4.B. For each of the symmetries, g, in the above group, find the least power, k, of g with g^k = identity. Use the results to decide if the group above is isomorphic to the group Z_4 with addition mod 4.

5.A. Let $G = Z_{18}$ with addition mod 18 and $H = \{0,3,6,9,12,15\}$. Find the sets $H+x = \{(h + x) \bmod 18 : h \text{ in } H\}$ for x = 1,2,3,4,5.

5.B. Suppose G is an unspecified group, H is a subgroup of G, and x is an unspecified value in G. For a,b in G, say a and b are related if $a*b^{-1}$ is in H*x. Show that this relation is an equivalence relation (ie symmetric, reflexive and transitive).

$H*x = \{ h*x : h \in H \}$.

6.A. For the following members a of the given groups, find a^2, a^{-1}, $(a^2)^{-1}$ and $(a^{-1})^2$:

i. a = (1 3 5) in S_3 with composition

ii. a = 2 in $Z_{11} - \{0\}$ with multiplication mod 11.

iii. A = 2 in Z_{11} with addition mod 11.

6.B. There are two possible interpretations of a^{-2} in a group. Namely a^{-2} is either $(a^2)^{-1}$ or $(a^{-1})^2$. Prove, using only the group properties, that $(a^2)^{-1} = (a^{-1})^2$.